HYDRODYNAMICAL STUDY OF ADVECTIVE ACCRETION FLOW AROUND NEUTRON STARS

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Here we study the accretion process around a neutron star, especially for the cases where shock does form in the accretion disk. In case of accretion flows around a black hole, close to the horizon matter is supersonic. On the other hand for the case of neutron stars and white dwarfs, matter must be subsonic close to the inner boundary. So the nature of the inflowing matter around neutron stars and white dwarfs are strictly different from that around black holes in the inner region of the disk. Here we discuss a few phenomena and the corresponding solutions of hydrodynamic equations of matter in an accretion disk around a slowly rotating neutron star without magnetic field.

1 Introduction

We study a few aspects of the accretion process around a neutron star. Chakrabarti [1,2] showed steady state solutions of the inflowing matter in advective accretion disk and made a bridge between the accretion and the wind solutions. In 1997, Chakrabarti and Sahu [3] revisited the Bondi type flows onto a neutron star. In all those studies the central compact object, namely the neutron star was chosen non-rotating or in the limit of very slow rotation. Apart from that, it was also assumed to be weakly magnetised. In this paper, we consider that the infalling matter possesses some angular momentum and thereby forms a disk. Hence our accretion process around neutron stars is more realistic than Bondi type flow.

Far away from the central compact object, the flow of the infalling matter does not depend on the nature of the central object, that is whether it is a black hole or a neutron star and therefore the solutions of the accretion disk are the same. As the matter comes closer to the central object, the inner boundary condition becomes important. In case of a neutron star, matter has to stop at the stellar surface, i.e., in the very inner region, speed of the flow must be subsonic. On the other hand if it is a black hole, due to the absence of a hard surface, in the inner region of the accretion disk the matter falls attaining a supersonic speed. This is the basic difference between the accretion flows around a black hole and a neutron star.

When the matter falls towards the compact object, the shock wave may form [1,3,4]. There are several kinds of shock formation, namely, Rankine-Hugoniot, isentropic and isothermal [5,6]. To form the shock wave in a flow, certain conditions have to be satisfied at the shock location according to the nature of the shock. In all kinds of shock formation, mass flux and total momentum of the matter are always conserved at the shock location. But there must be a jump in velocity (ϑ) of the matter. For the Rankine-Hugoniot shock, at the shock location, matter jumps discontinuously from the supersonic branch to the subsonic branch generating entropy,

keeping constant energy E of the flow. In the case of isentropic shock, the discontinuous parameter is energy at the shock location. In both the cases, temperature jumps up at the shock location. For the isothermal shock, because of strong radiative cooling, both energy and entropy jump discontinuously at the shock location. In this paper, with other discussions we will discuss about the formation of a shock in the accretion disk mainly around a neutron star. We will also study the general behaviour of the matter flow around a neutron star with or without shock and how it differs from that around a black hole.

Around a black hole only one stable shock formation is possible. There, after forming the shock, matter comes down from the supersonic branch to the subsonic branch and passing through the inner sonic point falls into the black hole. In the case of a neutron star, the formation of the shock in a particular matter flow (upto stellar surface) may happen twice in certain situations. For a neutron star, after the formation of a shock (exactly in the same way as for the black hole accretion), matter becomes subsonic. Then passing through the inner sonic point it attains supersonic speed. As close to the stellar surface the matter speed must be subsonic, another shock formation is necessary if the matter has to fall on the surface of the compact star. Thus, around a compact star with hard surface , if one shock forms, another shock has to form in the flow.

To study all the above mentioned properties of the infalling matter towards the compact object, we consider viscous flows with angular momentum. In the next section, we briefly discuss about the basic equations of the problem. In §3 we indicate the solution procedure and discuss about the possible solutions. In §4 we show solutions and finally in §5 draw our conclusions.

2   Basic Equations of the Flow

We use a well understood model of the advective accretion flow close to the black hole and the neutron star in the sub-Keplerian region of the disk [1]. According to our assumption, viscosity alone is responsible for the energy dissipation in the disk. We solve the equations given below to obtain the thermodynamic quantities.
(a) The continuity equation:

$$\frac{d}{dx}(\Sigma x \vartheta) = 0, \qquad (1a)$$

where, $\Sigma = h(x)\rho$ is the vertically integrated density.
(b) The radial momentum equation:

$$\vartheta \frac{d\vartheta}{dx} + \frac{1}{\rho}\frac{dP}{dx} + \frac{\lambda_{Kep}^2 - \lambda^2}{x^3} = 0. \qquad (1b)$$

Here, the first term represents advection which actually gives the information of kinetic energy of the infalling matter. The second term is due to the matter pressure and then last one is the combination of gravitational and centrifugal force terms.

(c) The azimuthal momentum equation:

$$\vartheta \frac{d\lambda(x)}{dx} - \frac{1}{\Sigma x} \frac{d}{dx}(x^2 W_{x\phi}) = 0, \qquad (1c)$$

with $W_{x\phi}$ is the vertically integrated viscous stress giving rise to the azimuthal pressure.

(d) The entropy equation:

$$\Sigma v T \frac{ds}{dx} = \frac{h(x)\vartheta}{\Gamma_3 - 1}\left(\frac{dp}{dx} - \Gamma_1 \frac{p}{\rho}\right) = Q^+ - Q^-$$

$$= f(\alpha, x, \dot{m})Q^+. \qquad (1d)$$

Here, Q^+ and Q^- are the viscous heat gained and lost by the flow respectively where for simplicity Q^- is chosen proportional to Q^+ with proportionality constant $g = 1 - f(\alpha, x, \dot{m})$, where $f(\alpha, x, \dot{m})$ is the cooling factor and $\dot{m}$ is the mass accretion rate in unit of the Eddington rate. The cooling is considered to be provided by bremsstrahlung and Comptonization effect. We use the standard definitions of Γ [7],

$$\Gamma_3 = 1 + \frac{\Gamma_1 - \beta}{4 - 3\beta}, \qquad (2a)$$

$$\Gamma_1 = \beta + \frac{(4 - 3\beta)^2(\gamma - 1)}{\beta + 12(\gamma - 1)(1 - \beta)} \qquad (2b)$$

and $\beta(x)$ is the ratio of gas pressure to total pressure as,

$$\beta(x) = \frac{\rho k T / \mu m_p}{\rho k T / \mu m_p + \bar{a}T^4/3}. \qquad (3)$$

Here, $\bar{a}$ is the Stefan's constant, k is the Boltzmann constant, m_p is the mass of the proton, μ is the mean molecular weight. Using the above definitions, eqn. (1d) becomes,

$$\frac{4 - 3\beta}{\Gamma_1 - \beta}\left[\frac{1}{T}\frac{dT}{dx} - \frac{1}{\beta}\frac{d\beta}{dx} - \frac{\Gamma_3 - 1}{\rho}\frac{d\rho}{dx}\right]$$

$$= f(\alpha, x, \dot{m})\frac{Q^+}{\vartheta P h(x)} = \frac{f\alpha x}{v}\frac{d\Omega}{dx}. \qquad (1e)$$

Here, we concentrate on the solutions with constant β. Actually, we study the relativistic flows where $\beta \sim 0$ and consequently $\Gamma_1 = \Gamma_3 = \frac{4}{3}$. Although very far away from the compact object, flows need not be relativistic (hence β need not be $\frac{4}{3}$), here for simplicity, we consider β as a constant throughout the particular cases. Similarly, we consider the cases for $f(\alpha, x, \dot{m}) = $ constant, though it is easy to understand that $f \sim 0$ in the Keplerian disk region and is greater than 0 near the compact object depending on the efficiency of cooling (governed by $\dot{m}$, for instance). We use the Paczyński-Wiita pseudo-potential [8] to describe the geometry of space using non-relativistic equations (as $\frac{\lambda_{Kep}^2}{x^3} = \frac{1}{2(x-1)^2}$). The half-thickness of the disk

$h(x) \sim ax^{1/2}(x-1)$ at a radial distance x is obtained from vertical equilibrium assumption, where a is the adiabatic sound speed $(a^2 = \gamma P/\rho)$, $\lambda(x)$ is the specific angular momentum, ϑ is the radial velocity, s is the entropy density of the flow. The constant α above is the Shakura-Sunyaev [9] viscosity parameter used to express stress tensor in terms of the total pressure Π due to the radial motion. We follow Chakrabarti [1,2] to do the calculations for total pressure, stress tensor $W_{x\phi}$, viscous heat generation which are not repeated here. From the continuity equation (Eqn. (1a)), we find the mass accretion rate to be given by

$$\dot{m} = 2\pi\rho h(x)\vartheta x. \tag{4}$$

Here, 2π is the geometric factor. The radial co-ordinate x is expressed in unit of Schwarzschild radius and the velocity is in unit of speed of light.

From the azimuthal momentum Eqn. (1c) we get,

$$\lambda - \lambda_{in} = \alpha\frac{x}{\vartheta}a^2\left(\frac{2}{3\gamma-1} + M^2\right), \tag{5}$$

where, $M = \vartheta/a$ is the Mach number of the flow and λ_{in} is the specific angular momentum at the inner edge of the flow.

3 Solution procedure and Discussion

From the given set of Eqns. 1(a-c,e) one can eliminate $\frac{da}{dx}$ and $\frac{d\rho}{dx}$ terms and finally obtain the equation of the form

$$\frac{d\vartheta}{dx} = \frac{F_1(x,a,\vartheta)}{F_2(\vartheta,a)}, \tag{6}$$

where,

$$F_1 = \left(\frac{\gamma+1}{a(\gamma-1)} - \frac{4a\alpha^2 f}{\vartheta^2(3\gamma-1)}\left(\frac{2}{3\gamma-1} + \frac{\vartheta^2}{a^2}\right)\right)\left(a\frac{5x-3}{2x(x-1)} + \frac{\gamma}{a}\left(\frac{l^2}{x^3} - \frac{1}{2(x-1)^2}\right)\right)$$

$$+ \frac{5x-3}{2x(x-1)} - \frac{\alpha f}{\vartheta}\left(\frac{2}{3\gamma-1} + \frac{\vartheta^2}{a^2}\right)\left(\alpha\vartheta\left(\frac{2}{3\gamma-1}\frac{a^2}{\vartheta^2} + 1\right) - \frac{2\lambda^2}{x^2}\right),$$

$$F_2 = \left(\frac{\gamma+1}{a(\gamma-1)} - \frac{4a\alpha^2 f}{\vartheta^2(3\gamma-1)}\left(\frac{2}{3\gamma-1} + \frac{\vartheta^2}{a^2}\right)\right)\left(-\frac{\gamma}{a}\left(\vartheta - \frac{a^2}{\gamma\vartheta}\right)\right) + \frac{1}{\vartheta}$$

$$- \frac{\alpha^2 f}{\vartheta}\left(\frac{2}{3\gamma-1} + \frac{\vartheta^2}{a^2}\right)\left(1 - \frac{2a^2}{(3\gamma-1)\vartheta^2}\right).$$

We shall concentrate upon the sub-Keplerian flow starting from the region where the flow deviates from Keplerian to sub-Keplerian. Here the flow could be of two kinds. In one case matter does not attain supersonic speed at all, thus there is no sonic point in this flow. In another case, matter passes through the sonic point and attains supersonic speed, then it takes again the subsonic branch (that is explained in later section) and falls onto the stellar surface.

For a complete run, we supply the basic parameters, namely, the location of the sonic point x_{cr}, the specific angular momentum at the inner edge of the flow λ_{in}, the polytropic index γ, the ratio f of advected viscous heat flux $Q^+ - Q^-$ to heat generation rate Q^+, the viscosity parameter α, the accretion rate $\dot{m}$ and the mass of the compact object in unit of solar mass. The derived quantities are: x_K where the Keplerian flow becomes sub-Keplerian, the ion temperature T, the flow density ρ, the radial velocity ϑ and the azimuthal momentum λ of the entire flow from x_K to the stellar surface. Far away from a compact object, the matter speed is very low (much lower than the sound speed), on the other hand, close to the compact object the matter speed may overcome the speed of sound. Thus, in that case, there is a location, namely *sonic point*, where this transition of the matter speed from the subsonic to the supersonic occurs. From the expression of F_2, it is very clear that at that location it is zero. Therefore, to have a continuous velocity gradient, F_1 must be zero at that radius. Thus, in the flow, if sonic points exist, at the sonic location both the numerator and denominator of $\frac{d\vartheta}{dx}$ vanish and giving two equations,

$F_1(x, a, \vartheta) = 0$ and $F_2(\vartheta, a) = 0$.

The equation $F_2(\vartheta, a) = 0$ simply gives the expression for Mach number (M) at the sonic point. As $F_1(x, a, \vartheta) = 0$ is quartic equation, in principle there will be four roots of x of which either all are real or two real and two complex or all are complex. In a particular flow, if real roots at all exist, first it should be checked whether its location is inside the '$x = 1$' surface (according to the Paczyński-Wiita potential) or not. If the root $x > 1$, sonic transition is possible for that flow. If there are two real physical roots, two possible sonic transitions exist. In earlier studies of the black hole accretion disk, it is seen that for existence of a single sonic point outside the horizon there is no shock formation [1]. After passing through that (inner) sonic point matter falls into the black hole. If there are two physical sonic points, first, passing through the outer sonic point matter will attain supersonic speed. Then if the shock condition is satisfied matter will jump from supersonic to subsonic branch and after passing through the inner sonic point it will fall into the black hole. On the other hand if the roots are complex then there is no solution of accretion flow for that particular parameter region having sonic point. But, there may be a solution of the accretion disk around neutron star without having any sonic point, as we have mentioned above that this is one kind of solution around the neutron star. There may be an another case, where after passing through the inner sonic point at the closer region to the compact star matter attains supersonic speed. If shock forms in the flow, at the shock location matter jumps from supersonic branch to subsonic branch and becoming subsonic falls onto the surface of neutron star. From the early works of Chakrabarti [1,2,4,6] and numerical simulations done by Molteni et al. [10,11,12] and Sponholz & Molteni [13] it was concluded that in the accretion disk around a black hole only one shock is possible. The formation of such a shock in accretion disk was also verified and studied by other independent groups [14,15,16,17]. Recently Das et al. [18] have analysed this shock in accretion disk analytically. The formation of shock strictly depends on whether the flow satisfies the shock conditions or not. If the Rankine-Hugoniot conditions are satisfied by the flow, matter always likes to jump from supersonic to subsonic branch which is

of higher entropy and hence the more stable branch at the same energy. The shock conditions are given below as [4]:

(a) conservation of energy at the shock location

$$E_+ = E_-, \tag{7a}$$

(b) conservation of the mass flux at the shock

$$\dot{m}_+ = \dot{m}_-, \tag{7b}$$

(c) momentum balance condition

$$W_+ + \Sigma_+ \vartheta_+^2 = W_- + \Sigma_- \vartheta_-^2 \tag{7c}$$

and
(d) generation of entropy

$$s_+ > s_-, \quad T_+ > T_-. \tag{7d}$$

Here, subscript '+' and '−' refer for quantity just after and before the shock respectively and W denotes the vertically integrated pressure as $W = \int_{-h}^{+h} P dx = 2PI_{n+1}h$, where $I_n = \frac{(2^n n!)^2}{(2n+1)!}$, n being the polytropic index and P is the pressure at the equatorial plane. Whenever the Rankine-Hugoniot shock forms and matter jumps from supersonic to subsonic branch, these four conditions strictly satisfy. Alternatively, we can say whenever the inflowing matter satisfies these four conditions, shock forms in the flow. In this present paper we will show that in the disk around a neutron star, formation of two shocks is very natural. Thus in a certain parameter region matter may change its branch (i.e., it will appear as transonic) twice in a particular flow. Sometimes it may happen that shock does not form in a flow when matter attains supersonic speed at far away ($\sim 50 - 90$ Schwarzschild radius) and continues to fall smoothly towards a black hole. This case is unstable, any disturbance created into the flow may cause a change of the matter from the supersonic to the subsonic branch via shock. For these unstable cases, at the very inner region of the disk (much inside than the radii $\sim 10 - 15$ where usually shock forms around black hole [1,19]) still a shock may form and the matter may jump from the supersonic branch to the subsonic branch and fall onto stellar surface. Here, it has to be noted that the location, where the shock conditions (here, according to Rankine-Hugoniot shock) satisfy for a particular flow, must be greater than the outer radius of the neutron star, otherwise there is no scope to form the shock. There is an another category of matter flow with shock. There, at around $10 - 15$ Schwarzschild radius one shock forms in a similar way as the shock formation in the black hole accretion disk. Then after the matter passes through the inner sonic point close to the neutron star's surface a second shock forms and becoming subsonic the matter falls onto the stellar surface (again the necessary condition is the outer radius of the star must be shorter than the shock location). When four real roots exist, it is seen that for any physical parameter set one of the root is always inside the horizon of the black hole or the outer surface of the star ($x = 1$) [4,6]. Out of the other three roots, two are of the 'X' type and other is of the 'O' type sonic point. We know that the 'O' type sonic point is not physical. In the case of a shock, sonic transitions occur through the 'X' type sonic points which are either
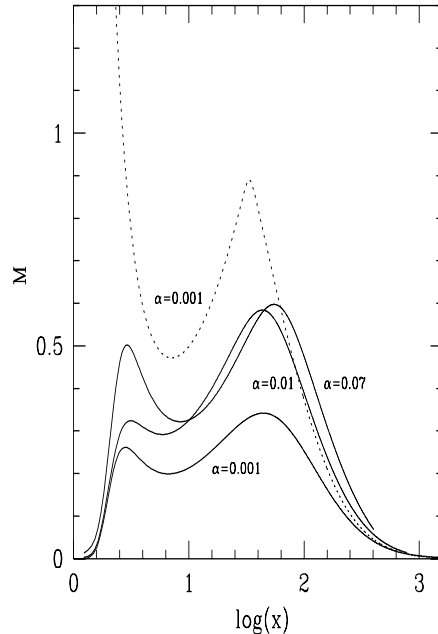
Fig. 1: Variation of Mach number M as a function of logarithmic radial distance. Solid curves are for neutron star and dotted one is drawn for accretion around black hole. For different values of α (are shown on each curve) variation changes. For complete set of parameters see TABLE-I. Here, no shock forms in the flow.

side of the 'O' type. So altogether three different cases might be occurred around a neutron star, they are: (1) no shock formation, (2) unstable one shock formation, (3) two shocks formation. As because, at most two physical sonic points may exist, maximum two shocks may occur in accretion flow around a neutron star.

4   Results

We have studied the accretion phenomena around a slowly rotating neutron star with weak magnetic field for different sets of physical parameters. Here we choose a few sets of suitable parameters to show the behaviour of matter flow for some interesting cases. Our main interest in this paper is to study the dynamical behaviour of the flow in a steady-state situation. Thus, following the standard practice, we will concentrate on to study the variation of the Mach number, from which it is easy to understand the nature of the matter speed; when it attains the supersonic speed and when it comes down to the subsonic branch.
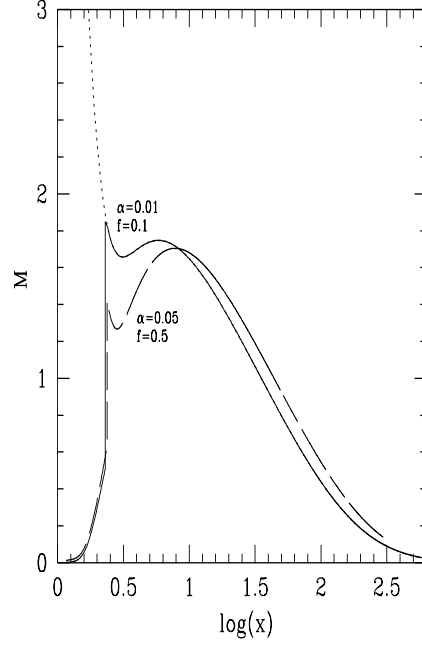
Fig. 2: Variation of Mach number M as a function of logarithmic radial distance. Solid and long dashed curves are for neutron star, dotted curve is drawn for accretion around black hole. For complete set of parameters see TABLE-II. Here, one shock is formed in the flow around neutron star.

4.1 CASE I

Here we choose the parameters in such a manner that the matter always stays in the subsonic branch. We list the parameters for three different cases in TABLE-I. We choose same $\dot{m}$, M_s (mass of the neutron star in the unit of solar mass) and f for all three cases but different viscosity parameter α. We have listed the values of sonic point x_{cr} for each case. Then we indicate the number of shock formed by 'S' and the corresponding shock location by 'x_s'.

TABLE-I

M_s	γ	x_{cr}	λ_{in}	α	$\dot{m}$	f	x_K	S	x_s
2	4/3	NO	1.6	0.07	1	0.1	401	0	NA
2	4/3	NO	1.6	0.01	1	0.1	783.7	0	NA
2	4/3	NO	1.65	0.001	1	0.1	1655.7	0	NA

It is seen from the TABLE-I that as α decreases x_K increases as well as residence time of the matter in the disk increases. In Fig. 1, we show the variation of Mach number with different values of α. As α decreases, energy momentum transfer rate decreases, as well as the corresponding Mach number and the centrifugal force decrease. Thus with the increase of α, incoming matter attains more speed as well as more centrifugal force in the outward direction (as there is quadratic dependence on velocity). Finally matter falls on to the surface of neutron star. For the comparison we also show one solution for the matter (of $\alpha = 0.001$) that falls into a black hole of mass $10 M_\odot$. In that case it is clear from Fig. 1 that passing through the sonic point $x_{cr} = 2.7945$, matter falls supersonically into the black hole.

4.2 CASE II

Here we choose the parameter in such a manner that close to the surface of the neutron star, shock forms. As we discussed in previous section, this case is slightly unstable as matter attains a supersonic speed at far away from the neutron star and continues to fall supersonically from $x \sim 50$ to just before the shock in the inner region of the disk. Usually in stable cases shock forms at a much greater radius as we will show in the next subsection (CASE III). Here, no shock forms at the outer radius and matter smoothly falls supersonically. But in the very inner region it satisfies the Rankine-Hugoniot shock conditions and jumps discontinuously from the supersonic branch to the subsonic branch and falls on to the star's surface. We list the parameters for two different cases in TABLE-II. We choose same $\dot{m}$, M_s for both the cases but different viscosity parameter α and cooling factor f. We also indicate the number of shocks formed (by S) and its location (x_s) as in CASE I. Two sonic points are given as x_{cr1} and x_{cr2}.

TABLE-II

M_s	γ	x_{cr1}	x_{cr2}	λ_{in}	α	$\dot{m}$	f	x_K	S	x_s
2	4/3	50	2.869	1.6	0.05	1	0.5	481.4	1	2.38
2	4/3	50	3.156	1.6	0.01	1	0.1	783.6	1	2.3

In Fig. 2 we show the variation of Mach number with logarithmic radial distance. For higher viscosity, matter is slowed down at the centrifugal pressure dominated region (centrifugal effect is maximum at $x \sim 3.2$) more strongly because of high rate of energy momentum transfer. It can be noted that if α is very high, say ≥ 0.1, centrifugal barrier may not appear because of very small x_K. At that low radius centrifugal force could not be comparable to the gravitational force, so the centrifugal pressure is smeared out. In Fig. 2, we see that the shock forms at very close to the neutron star for both the cases. The shock location is $\sim 2.3 - 2.4$ (see TABLE-II) and generally for the neutron star of mass $2 M_\odot$, outer radius is smaller than that shock location [20,21]. Thus the inner shock is possible here.

4.3 CASE III

Here we choose the parameter in such a manner that the shock forms twice in the flow around the neutron star. We have chosen two sets of parameters given in TABLE-III to describe this case. We choose the same accretion rate and mass of the star for both the cases. Two shock locations x_{s1} and x_{s2} are listed form on either side of the sonic points (given as x_{cr1} and x_{cr2}).

TABLE-III

M_s	γ	x_{cr1}	x_{cr2}	λ_{in}	α	$\dot{m}$	f	x_K	S	x_{s1}	x_{s2}
2	4/3	50	2.911	1.6	0.07	1	0.1	401	2	15.012	2.8025
2	4/3	50	2.869	1.6	0.05	1	0.5	481.4	2	13.9	2.737

During its motion, at first the matter satisfies the Rankine-Hugoniot shock conditions at a outer radius in comparison to the shock location of CASE II. The formation of the shock at this location is exactly similar to that which could be formed around a black hole. By this shock formation matter jumps from its unstable branch to a stable subsonic branch of higher entropy. This shock formation is independent of the central compact object, whether it is a black hole or a neutron star. After that, passing through the inner sonic point matter again attains a supersonic speed (see Fig. 3). Now another shock may form, depending on the nature of compact object. If it is a black hole there is no question of the second shock as the speed of the matter must be supersonic close to the horizon. On the other hand, if there is a neutron star, without having another shock matter can not reach the surface of the star in this present situation. This second shock occurs exactly in the similar way as the shock in CASE II, if the corresponding Rankine-Hugoniot shock conditions are satisfied. In Fig. 3, we show the formation of two shocks in the accretion flows around a neutron star. We show the Mach number variation of the accreting matter around neutron star for two different sets of initial parameter (solid and long-dashed curve). Two shock locations are denoted by sck1 and sck2. We also show the Mach number variation for the case if the matter with $\alpha = 0.07$ had fallen towards a black hole of mass $10 M_\odot$ by the dotted (stable flow with shock) and short-dashed (unstable flow without shock) curves. It is clear from the figure that upto just before the formation of the second shock, the solution around a black hole and a neutron star are similar; as if the matter does not know about the nature of central object. Thus, the formation of two shocks depends upon the nature of the compact object and the possibility of satisfaction of shock conditions. For certain choices of the flow parameter values, even for a neutron star the Rankine-Hugoniot shock conditions may not satisfy, though the matter attains a supersonic speed. For these cases, the matter will not reach the neutron star. Again it should be reminded that the inner shock location must be outside the radius of the neutron star. Here, as the radius of the shock location ($\sim 2.7 - 2.8$) is greater than the usual radius of a neutron star of mass $2 M_\odot$ [20,21], inner shock forms safely.

Here we also show the temperature variation in an accretion disk around a neutron star and a black hole in Fig. 4. Solid and long-dashed curves indicate the temperature profiles around a neutron star (indicated as NS in the figure) when viscosities of the infalling matter are 0.07 and 0.05 respectively (for other parameters, see TABLE-III). At the shock locations (as indicated by sck1 and sck2 in figure) temperature rises discontinuously. By the dotted and short-dashed curves we show the temperature variation of the inflowing matter of viscosity 0.07 when it falls towards a black hole of mass $10M_\odot$ (as indicated BH in the figure). Dotted curve is drawn for stable shock case and short-dashed curve is for unstable no-shock case (if the same matter had fallen without forming a shock). Although the virial temperature of the accretion disk may be very high as of the order of 10^{11}K, following Chakrabarti & Mukhopadhyay [22] and Mukhopadhyay & Chakrabarti [19] we have taken into account the cooling process so that the temperature reduces to of the order of 10^9K. As we consider the entire flow is relativistic, i.e., β is very low (radiation pressure dominated flow), the soft photon in the disk is very profuse. Thus, the virial temperature may be reduced by the inverse-Compton effect. As there is the possibility of the formation of two shocks in the disk around a neutron star temperature is still high enough. It is very clear from the above discussions that for the accretion around a neutron star, temperature and density are higher compared to that around a black hole.

5 Conclusions

In this paper we study the accretion flows around a neutron star having very slow rotation and weak magnetic field. We also compare the result with that around a black hole. We consider the recent concept of the accretion disk model and study the flow properties. We see that the shock may or may not form in the disk. We choose three different regions in the parameter space to show different features of the inflowing matter. We see, though in case of the accretion disk around a black hole only one shock is possible, around a neutron star formation of double shock is very natural, if the outer radius of the neutron star is not very very large (like ~ 3 Schwarzschild radius) which is unlikely. As the incoming matter is slowed down abruptly at the each shock location, the overall density becomes higher for a neutron star accretion disk compared to that for a black hole as two shocks may form around the neutron star. Similarly the temperature in the disk is comparatively higher, for a neutron star, as the temperature also jumps at the shock location. The higher density and higher temperature are not the result only of shock formation. Even if the shock does not form, the density and the temperature of the disk are higher close to the neutron star compared to the case of the black hole, because the matter has to be subsonic there. So, one can conclude that the accretion disk around a neutron star is very favourable for nucleosynthesis. Here it can be reminded that all along we have talked about the sub-Keplerian region of the accretion disk. In this context we can propose that the study of nucleosynthesis in the sub-Keplerian accretion disk around a neutron star should be done in future. We know that the advective accretion disk around a black hole is enough hot for nucleosynthesis [19,22,23,24,25], which are different from that of the star. As the density and temperature might be
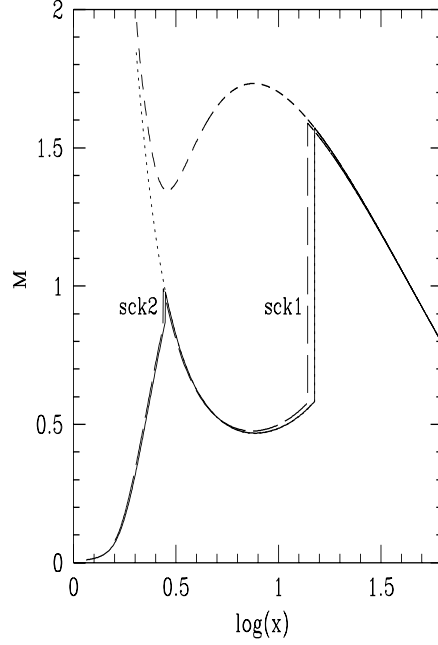
Fig. 3: Variation of Mach number M as a function of logarithmic radial distance. Solid and long-dashed curves are for neutron star and dotted (stable case) and short-dashed (unstable case) curves are drawn for accretion around black hole. Clearly two shocks form in the flow around neutron star which locations are indicated as sck1 and sck2. For complete set of parameters see TABLE-III.

higher around a neutron star, more efficient nucleosynthesis is expected close to it. Earlier we saw that the high temperature of accretion disk around a black hole is very favourable for the photo-dissociations and the proton capture reactions [19]. As the accretion disk around a neutron star is hotter than that around a black hole, even ^4He which has a high binding energy may dissociate into deuterium and then into proton and neutron. If we consider the accreting matter comes from the nearby 'Sun like' companion star, the initial abundance of ^4He in the accreting matter is about 25%. Therefore, by the dissociation of this ^4He, neutron may produce in a large scale which could give rise the neutron rich elements. Guessoum & Kazanas [26] showed that the profuse neutron may produce in the accretion disk and through the spallation reactions Li may be produced in the atmosphere of the star. When the neutron comes out from the accretion disk by the formation of an outflow, in that comparatively cold environment ^7Li may be produced, which can be detected on the stellar surface. Earlier it was shown that the metalicity of the galaxy may be influenced when outflows are formed in the hot accretion disk around black holes [19]. In case of the lighter galaxy, the average abundance of the isotopes of Ca, Cr may significantly change. Also the abundance of lighter elements, like the isotopes
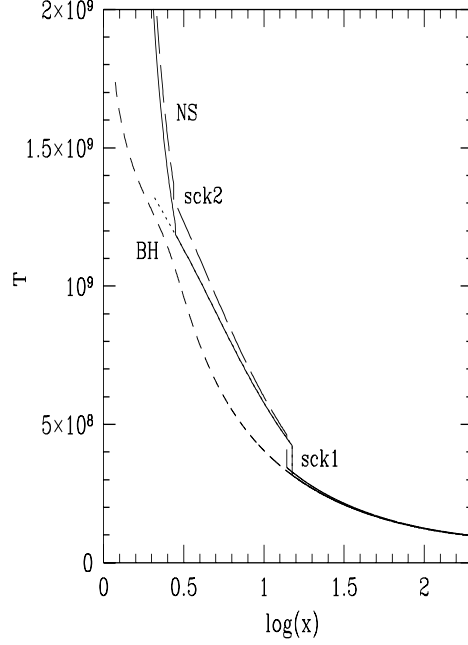
Fig. 4: Variation of temperature in K as a function of logarithmic radial distance. By NS and BH we mean the temperature distribution around neutron star and black hole respectively. We indicate the shock locations by sck1 and sck2.

of C, O, Ne, Si etc. may be increased significantly. As the temperature of the accretion disk around a neutron star is higher, the expected change of abundance of these elements and the corresponding influence on the metalicity of the galaxy is higher. We would like to pursue all these studies in the next work.

Molteni and his coworkers have already shown that the oscillatory nature of the shock location is related to the cooling and advective time scale of the matter in the disk [12,27]. Corresponding oscillation frequency is related to the location of the shock and the observed QPO frequency for the various black hole candidates can be explained. We know that the QPO frequencies are observed mainly in two ranges: Hz and KHz [28,29]. The frequency depends on the time taken by the inflowing matter to reach the compact object from the shock location. From the discussion of Molteni, Sponholtz & Chakrabarti [12], it is clear that as the shock location comes closer to the compact object QPO frequency increases. According to the earlier study of the accretion flow around a black hole [1], it was found that the shock may form at a significantly outer radius. Following the prescription of Molteni, Sponholtz & Chakrabarti [12], we can find the corresponding QPO frequency of the order of Hz. Those theoretical calculations match with the observed QPO frequencies, say for the candidates GS 339 4 and 1124 68. Thus, these lower QPO frequencies can be explained when outer shocks are formed. Now following the

same prescription given by Molteni, Sponholtz & Chakrabarti [12], the higher QPO frequencies can also be explained for the case of a neutron star accretion disk, because of the inner shock around a neutron star. As for the presence of an inner shock, matter takes less time to fall onto the stellar surface from the shock location, corresponding frequencies will be higher. Therefore for the explanation of the KHz QPO frequency, the double shock formation in the accretion disk around a neutron star is very significant. Our next step of the study will be the theoretical calculation of the KHz QPO frequency and to compare it with the observed values.

Acknowledgments

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